3rd Edition

Precalculus
A Right Triangle Approach

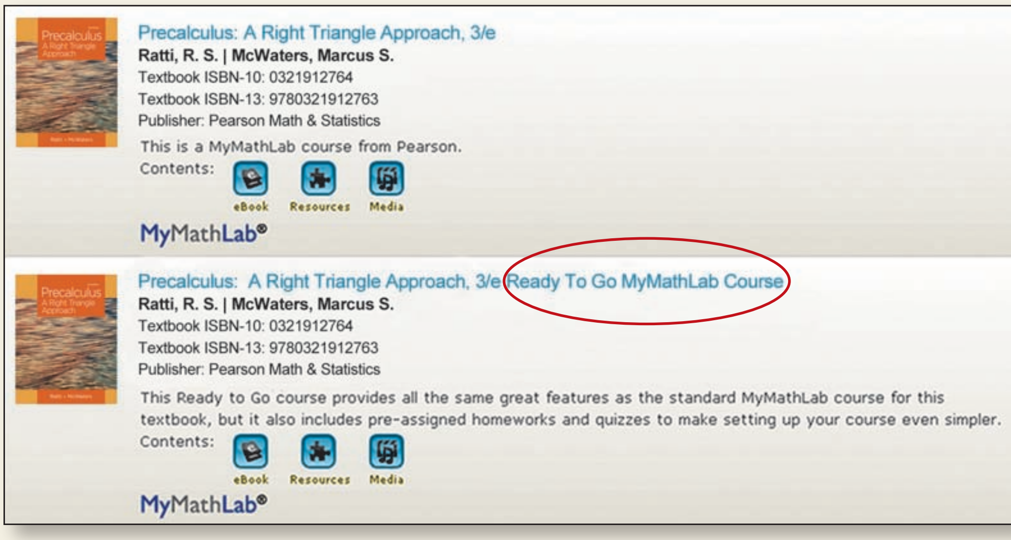
Ratti ▪ McWaters

MyMathLab® Ready to Go!

It's easier than ever to get started with MyMathLab! Students benefit from immediate feedback on homework assignments, additional practice, and review resources—all aimed at improving their results.

Quick & Easy

Assignments and quizzes are prebuilt, making it easy to get started with MyMathLab. When creating your course, simply choose the Ready to Go version.



Author Selected

The author has chosen homework assignments, quizzes, and tests that will help students get the most out of learning through online assessments. You have the flexibility to edit, change, and add to the Ready to Go assignments at any time.

Getting Ready

Skill review quizzes are integrated throughout the Ready to Go MyMathLab course, testing students on prerequisite knowledge. Students receive personalized, just-in-time review assignments, based on quiz results, allowing for additional practice on forgotten concepts.

Successful Students

MyMathLab improves the quality of learning in any environment—lab-based, hybrid, fully online, and traditional classroom settings. Implementing and requiring MyMathLab leads to quantifiable results in student retention and success.

Scan the code or visit
www.mymathlab.com/success-stories
for a growing list of case studies.

Achieve Your Potential

The authors J.S. Ratti and Marcus McWaters, have developed specific content in MyMathLab® to ensure you have many resources to help you achieve success in mathematics—and beyond!

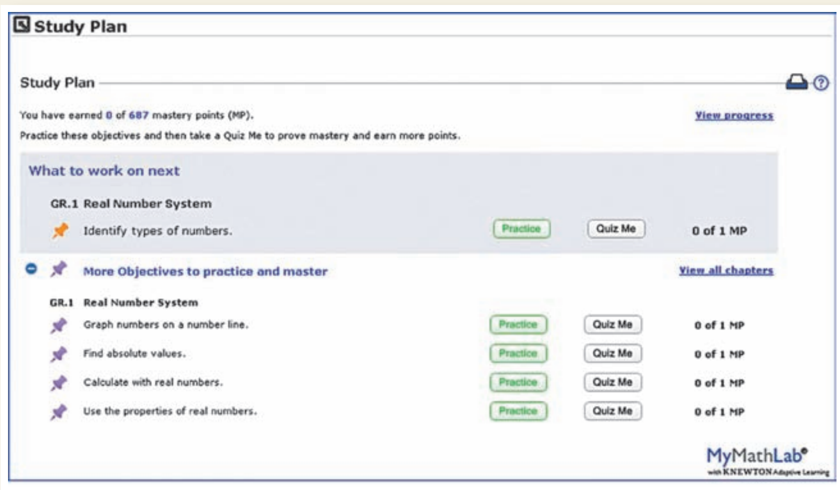
The MyMathLab features described here will help you:

- **Brush up** on math skills and concepts you may have forgotten.
- **Retain** new concepts as you move through your math course.
- **Develop** skills that will help with your transition to college and to your profession.

Adaptive Study Plan

The Study Plan will help you study more efficiently and effectively.

Your performance and activity are assessed continually in real time, providing a personalized experience based on your individual needs.



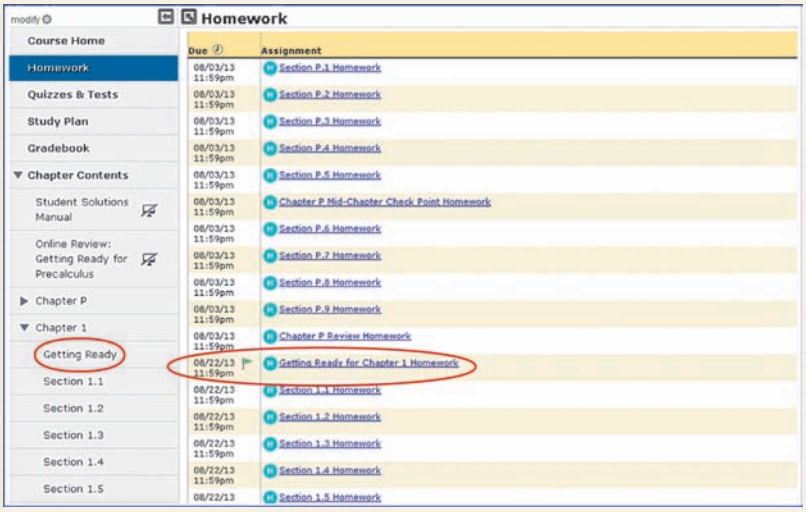
Skills for Success

The Skills for Success Module supports your continued success in college.

This module provides tutorials and guidance on a variety of topics, including the transition to college, online learning, time management, and more.

Additional content is provided to help with the development of professional skills such as resume writing and creation and interview preparation.

Getting Ready



Are you frustrated when you know you learned a math concept in the past but you can't quite remember the skill when it's time to use it? Don't worry!

The authors have included Getting Ready material so that you can brush up on forgotten material efficiently by taking a quick skill review quiz to pinpoint the areas in which you need help.

Then a personalized homework assignment provides additional practice on those forgotten concepts, right when you need it.

Maintaining Skills

As you work through your math course, these MyMathLab® exercises support ongoing review to help you maintain essential skills.

The ability to recall important math concepts as you continually acquire new mathematical skills will help you be successful in this math course and in your future math courses.

Scan the code or visit **www.mymathlab.com/success-stories** to learn how students across the country have used MyMathLab to achieve their potential and be successful in their math courses.

Precalculus: A Right Triangle Approach

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Precalculus: A Right Triangle Approach

Third Edition

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University of South Florida

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To Our Wives,

Lata and Debra

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FOREWORD

We're pleased to present this new edition of our text for Precalculus: A Right Triangle Approach. Our experience teaching this material at the University of South Florida has been exceptionally rewarding. Because students are accustomed to information being delivered by electronic media, the introduction of MyMathLab® into our courses was seamless. We hope you will have a similar experience.

Many challenges face today's precalculus students and instructors. Students arrive with various levels of comprehension from their previous courses. Instead of really learning the concepts presented, students often resort to memorization to pass the course. As a result, a course needs to get students to a common starting point and engage them in becoming active learners, without sacrificing the solid mathematics essential for conceptual understanding. Instructors in this course are faced with the task of providing students with an understanding of precalculus, preparing them for the next step, and ensuring that they find mathematics useful and interesting. Our efforts, however, have been aided considerably by the many suggestions we have received from users of the first and second editions of this text.

In this text, there is a strong emphasis on both concept development and real-life applications. Topics such as functions, graphing, the difference quotient, and limiting processes provide thorough preparation for the study of calculus and will improve students' comprehension of algebra. Just-in-time review throughout the text ensures that all students are brought to the same level before being introduced to new concepts. Numerous applications are used to motivate students to apply the concepts and skills they learn in precalculus to other courses, including the physical and biological sciences, engineering, economics, and to on-the-job and everyday problem solving. Students are given ample opportunities throughout this course to think about important mathematical ideas and to practice and apply algebraic skills.

Throughout the text, we emphasize why the material being covered is important and how it can be applied. By thoroughly developing mathematical concepts with clearly defined terminology, students see the "why" behind those concepts, paving the way for a deeper understanding, better retention, less reliance on rote memorization, and ultimately more success. The level of exposition was selected so that the material is accessible to students and provides them with an opportunity to grow.

It is our hope that once you have read through our text, you will see that we were able to fulfill our initial goals of writing for today's students and for you, the instructor.

(Marcus McWaters)

(J. S. Ratti)

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PREFACE

Students begin precalculus classes with widely varying backgrounds. Some haven't taken a math course in several years and may need to spend time reviewing prerequisite topics, while others are ready to jump right into new and challenging material. In Chapter P and in some of the early sections of other chapters, we have provided review material in such a way that it can be used or omitted as appropriate for your course. In addition, students may follow several paths after completing a precalculus course. Many will continue their study of mathematics in courses such as trigonometry, finite mathematics, statistics, and calculus. For others, precalculus may be their last mathematics course.

Responding to the current and future needs of all of these students was essential in creating this course. We introduce each exercise set with several concept exercises. These exercises consist of fill-in-the-blank and true–false exercises. They are not computation-reliant, but rather test whether students have absorbed the basic concepts and vocabulary of the section. Exercises asking students to extrapolate information from a given graph now appear in much greater number and depth throughout the course. We continue to present our content in a systematic way that illustrates how to study and what to review. We believe that if students use this material well, they will succeed in this course.

New To The Third Edition

EXERCISES

- We continue to improve the balance of exercises and have added more challenging exercises.
- Answers to the Practice Problems have been moved to the end of the section, just before the exercises.
- New **Maintaining Skills** exercises, placed at the end of each exercise set, help refresh important concepts learned in previous chapters and allow for practice of skills needed for more advanced concepts in upcoming chapters.
- Overall, approximately 20% of the exercises have been updated, and we've added over 1000 additional exercises.

CHAPTER 1

- Applications of linear equations have been relocated in Section 1.
- A separate section "Complex Numbers: Quadratic Equations with Complex Solutions" introduces complex numbers and discusses complex roots of quadratic equations.
- Polynomial and rational inequalities are now discussed in Section 5.

CHAPTER 2

- Discussion of the domain of composite functions has been expanded and includes more examples.

CHAPTER 3

- New examples of quadratic functions have been included.
- The procedure for graphing rational functions has been rewritten to identify the relation of the graph to horizontal and oblique asymptotes.

CHAPTER 4

- The number e is now introduced in Section 1.
- A new section on logarithmic scales has been added.

CHAPTER 7

- The ambiguous case is now included in Section 7.1.
- Section 7.3 has been expanded to include the areas of regular polygons.

CHAPTER 8

- "Partial-Fraction Decomposition" has been moved to immediately follow Section 2, "Systems of Linear Equations in Three Variables."

Features

CHAPTER OPENER Each chapter opener includes a description of applications relevant to the content of the chapter and the list of topics that will be covered in the chapter. In one page, students see what they are going to learn and why they are learning it.

APPLICATION The discussion in each section begins with a motivating anecdote or piece of information that is tied to an application problem. This problem is solved in an example later in the section, using the mathematics covered in the section.

SECTION OPENERS The application section openers lend continuity to the section and its content, utilizing material from a variety of fields: the physical and biological sciences (including health sciences), economics, art and architecture, the history of mathematics, and more. Of special interest are contemporary topics such as the greenhouse effect and global warming, CAT scans, and computer graphics.

REVIEW On the first page of each section is a list of topics that students should review prior to starting the chapter. Section references with page numbers accompany the suggested review material so that students can readily find the material. The **Objectives** of the section are then clearly stated and numbered. Each numbered objective is paired with a similarly numbered subsection so that students can quickly find the section material for an objective.

DEFINITIONS AND THEOREMS All are boxed for emphasis and titled for ease of reference, as are lists of rules and properties.

FIGURES All figures are titled to make it easy to identify what is being illustrated.

EXAMPLES The examples include a wide range of computational, conceptual, and modern applied problems that are carefully selected to build confidence, competency, and understanding. Every example has a title that indicates its purpose. For every example, clarifying side comments are provided for each step in the detailed solution.

PRACTICE PROBLEM All examples are followed by a Practice Problem for students to try so that they can check their understanding of the concept covered. The answers to these problems are placed at the end of each section, just before the exercises.

PROCEDURE BOXES These boxes, interspersed throughout the text, present important procedures in numbered steps. Special Procedure in Action boxes present important multistep procedures, such as the steps for doing synthetic division, in a two-column format. The steps of the procedure are given in the left column, and an example is worked, following these steps, in the right column. This approach provides students with a clear model with which they can compare when encountering difficulty in their work. These boxes are a part of the numbered examples.

SUMMARY OF MAIN FACTS These boxes summarize information related to equations and their graphs, such as those of the conic sections.

HISTORICAL NOTES When appropriate, historical notes appear in the margin, giving students information on key people or ideas in the history and development of mathematics. This information is included to add flavor to the subject matter.

TECHNOLOGY CONNECTIONS Although the use of graphing calculators is optional in this course, Technology Connections give students tips on using calculators to solve problems, check answers, and reinforce concepts.

WARNINGS These boxes appear as appropriate throughout the text to let students know of common errors and pitfalls that can trip them up in their thinking or calculations.

RECALL Located in the margins, Recall Notes periodically remind students of a key idea they learned earlier in the text that will help them with the new problems at hand.

SIDE NOTES Students are given hints for handling newly introduced concepts.

DO YOU KNOW? These marginal notes provide students with additional interesting information on nonessential topics to keep them engaged in the mathematics presented.

EXERCISES The heart of any textbook is its exercises. Knowing this, we made sure that the quantity, quality, and variety of exercises meet the needs of all students. The problems in each exercise set are carefully graded to strengthen the skills developed in the associated section. Exercises are divided into three categories: **Basic Skills and Concepts** (developing fundamental skills), **Applying the Concepts** (using the section's material to solve real-world problems), and **Beyond the Basics** (providing more challenging exercises that give students an opportunity to reach beyond the material covered in the section). Exercises are paired so that the even-numbered Basic Skills and Concepts exercises closely follow the preceding odd-numbered exercises. All application exercises are titled and relevant to the topics of the section. Basic Skills and Concepts and Applying the Concepts exercises are intended for a typical student, whereas the Beyond the Basics exercises, generally more theoretical in nature, are suitable for honors students, special assignments, or extra credit. **Critical Thinking/Discussion/Writing** exercises appear in each exercise set and are designed to develop students' higher-level thinking skills and to reinforce and extend comprehension of the material. Calculator problems are included where appropriate.

END-OF-CHAPTER The chapter-ending material includes a **Summary of Definitions**, **Concepts**, **and Formulas**; **Review Exercises**; and **two Practice Tests**. The chapter summary, a brief description of key topics indicating where the material occurs in the text, encourages students to reread sections rather than memorize definitions out of context. The Review Exercises provide students with an opportunity to practice what they have learned in the chapter. (Students should then be prepared for Practice Test A in the usual open-ended format and Practice Test B, covering the same topics, in multiple-choice format.) All tests are designed to increase student comprehension and verify that students have mastered the skills and concepts in the chapter. Mastery of these materials should indicate a true comprehension of the chapter and the likelihood of success on the associated in-class examination.

THE NEW MAINTAINING SKILLS exercises, placed at the end of each exercise set, help refresh important concepts learned in previous chapters and allow for practice of skills needed for more advanced concepts in upcoming chapters.

CUMULATIVE REVIEW EXERCISES Starting with Chapter 2, these exercises appear at the end of every chapter to remind students that mathematics is not modular and that what is learned in the first part of the book will be useful in later parts of the book and on the final examination.

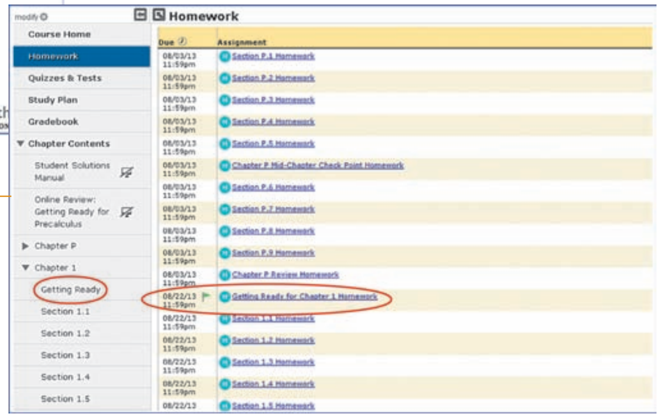
RESOURCES FOR SUCCESS

MyMathLab® Online Course (Access Code Required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from an **experienced partner** with educational expertise and an eye on the future. MyMathLab helps prepare students and gets them thinking more conceptually and visually through the following features:

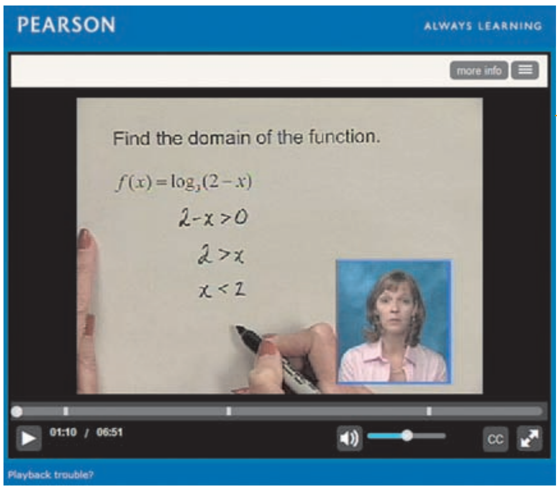
ADAPTIVE STUDY PLAN

The Study Plan makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized content–reinforcing concepts that target each student's strengths and weaknesses.



GETTING READY

Students refresh prerequisite topics through assignable skill review quizzes and personalized homework integrated in MyMathLab.

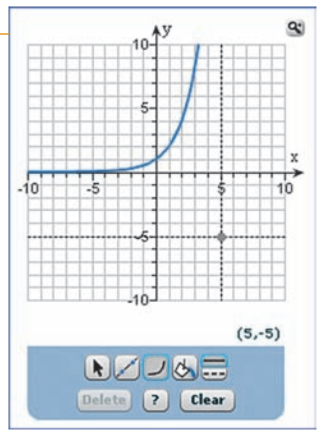


VIDEO ASSESSMENT

Video assessment is tied to key Example Solution videos to check student's conceptual understanding of important math concepts.

ENHANCED GRAPHING FUNCTIONALITY

Functionality allows graphing of 3-point quadratic functions, 4-point cubic graphs, and transformations in exercises.



SKILLS FOR SUCCESS MODULE in MyMathLab helps students succeed in collegiate courses and prepare for future professions.

MAINTAINING SKILLS exercises help refresh important concepts and allows for practice of skills needed for more advanced topics. These exercises are also assignable in MyMathLab.

Instructor Resources

Additional resources can be downloaded from **www.pearsonhighered.com** or hardcopy resources can be ordered from your sales representative.

READY TO GO MYMATHLAB® COURSE

Now it is even easier to get started with MyMathLab. The Ready to Go MyMathLab course option includes author-chosen preassigned homework, integrated review, and more.

TESTGEN®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

POWERPOINT® LECTURE SLIDES

Fully editable slides that correlate to the textbook.

ANNOTATED INSTRUCTOR'S EDITION

This version of the text includes answers to all exercises presented in the book. Sample homework assignments, selected by the authors, are annotated with an underline.

INSTRUCTOR'S SOLUTIONS MANUAL

Includes fully worked solutions to all textbook exercises.

Student Resources

Additional resources to help student success.

VIDEO LECTURES

- Section Summary videos cover key definitions and procedures for most sections. Example Solution videos walk students through the detailed solution process for many examples in the textbook.
- There are over 20 hours of video instruction specifically filmed for this book, making it ideal for distance learning or supplemental instruction on a home computer or in a campus computer lab.
- Videos include optional subtitles in English and Spanish.

STUDENT'S SOLUTIONS MANUAL

Provides detailed worked-out solutions to odd-numbered exercises.

ACKNOWLEDGMENTS

We would like to express our gratitude to the reviewers of this and previous editions, who provided such invaluable insights and comments. Their contributions helped shape the development of the text and carry out the vision stated in the preface.

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We invite all who use this book to send suggestions for improvements to Marcus McWaters at mmm@cas.usf.edu.

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CHAPTER P

Basic Concepts of Algebra

TOPICS

Many fascinating patterns in human and natural processes can be described in the language of algebra. We investigate events ranging from chirping crickets to the behavior of falling objects.

SECTION **P.1**

The Real Numbers and Their Properties

BEFORE STARTING THIS SECTION, REVIEW FROM YOUR PREVIOUS MATHEMATICS TEXTS

1. Arithmetic of signed numbers
2. Arithmetic of fractions
3. Long division involving integers
4. Decimals
5. Arithmetic of real numbers

OBJECTIVES

1. Classify sets of real numbers.
2. Use the ordering of the real numbers.
3. Specify sets of numbers in roster or set-builder notation.
4. Use interval notation.
5. Relate absolute value and distance on the real number line.
6. Identify the order of operations in arithmetic expressions.
7. Identify and use properties of real numbers.
8. Evaluate algebraic expressions.

Cricket Chirps and Temperature

Crickets are sensitive to changes in air temperature; their chirps speed up as the temperature gets warmer and slow down as it gets cooler. It is possible to use the chirps of the male snowy tree cricket (*Oecanthus fultoni*), common throughout the United States, to gauge temperature. (The insect is found in every U.S. state except Hawaii, Alaska, Montana, and Florida.) By counting the chirps of this cricket, which lives in bushes a few feet from the ground, you can gauge temperature. Snowy tree crickets are more accurate than most cricket species; their chirps are slow enough to count, and they synchronize their singing. To convert cricket chirps to degrees Fahrenheit, count the number of chirps in 14 seconds and then add 40 to get the temperature. To convert cricket chirps to degrees Celsius, count the number of chirps in 25 seconds, divide by 3, and then add 4 to get the temperature. In Example 10, we evaluate algebraic expressions to learn the temperature from the number of cricket chirps.

1 Classify sets of real numbers.

Classifying Numbers

In algebra, we use letters such as a, b, x, y, and so on, to represent numbers. A letter that is used to represent one or more numbers is called a **variable**. A **constant** is a specific number such as 3 or $\frac{1}{2}$ or a letter that represents a fixed (but not necessarily specified) number. Physicists use the letter c as a constant to represent the speed of light ($c \approx 300{,}000{,}000$ meters per second).

We use two variables, a and b, to denote the results of the operations of addition $(a + b)$, subtraction $(a - b)$, multiplication $(a \times b$ or $a \cdot b)$, and division $\left(a \div b \text{ or } \frac{a}{b}\right)$. These operations are called **binary operations** because each is performed on two numbers.

SIDE **NOTE**

Here is one difficulty with attempting to divide by 0: If, for example, $\frac{5}{0} = a$, then $5 = a \cdot 0$. However, $a \cdot 0 = 0$ for all numbers a. So we would have $5 = 0$; this contradiction means that there is no appropriate choice for $\frac{5}{0}$.

We frequently omit the multiplication sign when writing a product involving two variables (or a constant and a variable) so that $a \cdot b$ and ab indicate the same product. Both a and b are called **factors** in the product $a \cdot b$. This is a good time to recall that we never divide by zero. For $\frac{a}{b}$ to represent a real number, b cannot be zero.

Equality of Numbers

The **equal sign**, $=$, is used much like we use the word *is* in English. The equal sign means that the number or expression on the left side is equal or equivalent to the number or expression on the right side. We write $a \neq b$ to indicate that a is not equal to b.

Classifying Sets of Numbers

The idea of a set is familiar to us. We regularly refer to "a set of baseball cards," a "set of CDs," or "a set of dishes." In mathematics, as in everyday life, a **set** is a collection of objects. The objects in the set are called the **elements**, or **members**, of the set. In the study of algebra, we are interested primarily in sets of numbers.

In listing the elements of a set, it is customary to enclose the listed elements in braces, $\{ \}$, and separate them with commas.

We distinguish among various sets of numbers.

The numbers we use to count with constitute the set of **natural numbers**: $\{1, 2, 3, 4, \ldots\}$.

The three dots . . . may be read as "and so on" and indicate that the pattern continues indefinitely.

The **whole numbers** are formed by including the number 0 with the natural numbers to obtain the set $\{0, 1, 2, 3, 4, \ldots\}$.

The **integers** consist of the set of natural numbers together with their opposites and 0: $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.

Rational Numbers

The **rational numbers** consist of all numbers that *can* be expressed as the quotient or ratio, $\frac{a}{b}$ of two integers, where $b \neq 0$.

Examples of rational numbers are $\frac{1}{2}, \frac{5}{3}, \frac{-4}{17}$, and $0.07 = \frac{7}{100}$. Any integer a can be expressed as the quotient of two integers by writing $a = \frac{a}{1}$. Consequently, every integer is also a rational number. In particular, 0 is a rational number because $0 = \frac{0}{1}$.

The rational number $\frac{a}{b}$ can be written as a decimal by using long division. When any integer a is divided by an integer b, $b \neq 0$, the result is always a **terminating decimal** such as $\left(\frac{1}{2} = 0.5\right)$ or a **nonterminating repeating decimal** such as $\left(\frac{2}{3} = 0.666\ldots\right)$.

We sometimes place a bar over the repeating digits in a nonterminating repeating decimal. Thus, $\frac{2}{3} = 0.666\ldots = 0.\overline{6}$ and $\frac{141}{110} = 1.2818181\ldots = 1.2\overline{81}$.

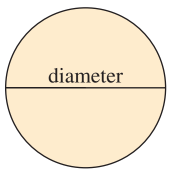
EXAMPLE 1 **Converting Decimal Rationals to a Quotient**

Write the rational number $7.\overline{45}$ as the ratio of two integers in lowest terms.

Solution

Let $x = 7.454545\ldots.$ Then

$$\begin{aligned} 100x &= 745.4545\ldots && \text{Multiply both sides by 100.} \\ \text{and} \quad x &= 7.4545\ldots && \\ \hline 99x &= 738 && \text{Subtract } x \text{ from } 100x. \\ x &= \frac{738}{99} && \text{Divide both sides by 99.} \\ &= \frac{82 \times \cancel{9}}{11 \times \cancel{9}} && \text{Common factor} \\ x &= \frac{82}{11} && \text{Reduce to lowest terms.} \end{aligned}$$



$\pi = \dfrac{\text{circumference}}{\text{diameter}}$

FIGURE P.1 Definition of π

Practice Problem 1 Repeat Example 1 for $2.132132132\ldots.$

Irrational Numbers

An **irrational number** is a number that cannot be written as a ratio of two integers. This means that its decimal representation must be nonrepeating and nonterminating. We can construct such a decimal using only the digits 0 and 1, such as 0.01001000100001. . . . Because each group of zeros contains one more zero than the previous group, no group of digits repeats. Other numbers such as π ("pi", see Figure P.1) and $\sqrt{2}$ (the square root of 2, see Figure P.2) can also be expressed as decimals that neither terminate nor repeat; so they are irrational numbers as well. We can obtain an approximation of an irrational number by using an initial portion of its decimal representation. For example, we can write $\pi \approx 3.14159$ or $\sqrt{2} \approx 1.41421$, where the symbol $\approx$ is read "is approximately equal to."

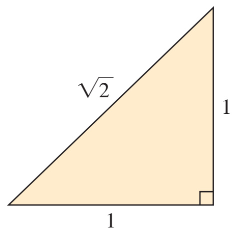


FIGURE P.2

No familiar process, such as long division, is available for obtaining the decimal representation of an irrational number. However, your calculator can provide a useful approximation for irrational numbers such as $\sqrt{2}$. (Try it!) Because a calculator displays a fixed number of decimal places, it gives a **rational approximation** of an irrational number.

It is usually not easy to determine whether a specific number is irrational. One helpful fact in this regard is that *the square root of any natural number that is not a perfect square is irrational.* So $\sqrt{6}$ is irrational but $\sqrt{16} = \sqrt{4^2} = 4$ is rational.

RECALL

An integer is a *perfect square* if it is a product $a \cdot a$, where a is an integer. For example, $9 = 3 \cdot 3$ is a perfect square.

Because rational numbers have decimal representations that either terminate or repeat, whereas irrational numbers do not have such representations, *no number is both rational and irrational.*

The rational numbers together with the irrational numbers form the **real numbers**. The diagram in Figure P.3 shows how various sets of numbers are related. For example, every natural number is also a whole number, an integer, a rational number, and a real number.

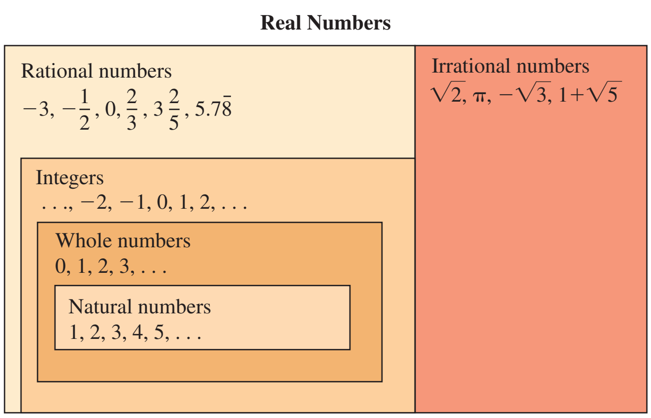


FIGURE P.3 Relationships among sets of real numbers

2 Use the ordering of the real numbers.

The Real Number Line

We associate the real numbers with points on a geometric line (imagined to be extended indefinitely in both directions) in such a way that each real number corresponds to exactly one point and each point corresponds to exactly one real number. The point is called the **graph** of the corresponding real number, and the real number is called the **coordinate** of the point. By agreement, ***positive numbers*** lie to the right of the point corresponding to 0 and ***negative numbers*** lie to the left of 0. See Figure P.4.

Notice that $\frac{1}{2}$ and $-\frac{1}{2}$, 2 and -2, and π and $-\pi$ correspond to pairs of points exactly the same distance from 0 but on opposite sides of 0.

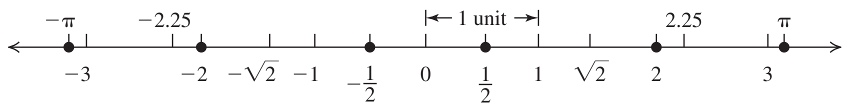


FIGURE P.4 The real number line

When coordinates have been assigned to points on a line in the manner just described, the line is called a **real number line**, a **coordinate line**, a **real line**, or simply a **number line**. The point corresponding to 0 is called the **origin**.

Inequalities

The real numbers are **ordered** by their size. We say that ***a* is less than *b*** and write $a < b$, provided that $b = a + c$ for some ***positive*** number c. We also write $b > a$, meaning the same thing as $a < b$, and say that ***b* is greater than *a***. On the real line, the numbers increase from left to right. Consequently, ***a is to the left of b on the number line when a < b***. Similarly, a is to the right of b on the number line when $a > b$. We sometimes want to indicate that at least one of two conditions is correct: Either $a < b$ or $a = b$. In this case, we write $a \le b$ or $b \ge a$. The four symbols $<, >, \le$, and $\ge$ are called **inequality symbols**.

EXAMPLE 2 **Identifying Inequalities**

Decide whether each of the following is true or false.

a. $5 > 0$ **b.** $-2 < -3$ **c.** $2 \le 3$ **d.** $4 \le 4$

SIDE NOTE

Notice that the inequality sign always points to the smaller number.

$2 < 7$, 2 is smaller.
$5 > 1$, 1 is smaller.

Solution

a. $5 > 0$ is true because 5 is to the right of 0 on the number line. See Figure P.5.

b. $-2 < -3$ is false because -2 is to the right of -3 on the number line.

c. $2 \le 3$ is true because 2 is to the left of 3 on the number line. (Recall that $2 \le 3$ is true if either $2 < 3$ or $2 = 3$.)

d. $4 \le 4$ is true because $4 \le 4$ is true if either $4 < 4$ or $4 = 4$.

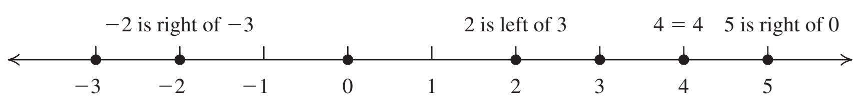


FIGURE P.5

Practice Problem 2 Decide whether each of the following is true or false.

a. $-2 < 0$ **b.** $5 \le 7$ **c.** $-4 > -1$

The following properties of inequalities for real numbers are used throughout this text.

Trichotomy Property: Exactly one of the following is true:

$$a < b, a = b, \text{ or } a > b$$

Transitive Property: If $a < b$ and $b < c$, then $a < c$.

The trichotomy property says that if two real numbers are not equal, then one is larger than the other. The transitive property says that "less than" works like "smaller than" or "lighter than." Frequently, we read $a > 0$ as "a is positive" instead of "a is greater than 0." We can also read $a < 0$ as "a is negative." If $a \geq 0$, then either $a > 0$ or $a = 0$, and we may say that "a is nonnegative."

3 Specify sets of numbers in roster or set-builder notation.

Sets

To specify a set, we do one of the following:

1. List the elements of the set (**roster method**)

2. Describe the elements of the set (often using **set-builder notation**)

Variables are helpful in describing sets when we use set-builder notation. The notation $\{x | x \text{ is a natural number less than } 6\}$ is in set-builder notation and describes the set $\{1, 2, 3, 4, 5\}$ using the roster method.

We read $\{x | x \text{ is a natural number less than } 6\}$ as "the set of all x such that x is a natural number less than six." Generally, $\{x | x \text{ has property } P\}$ designates the set of all x such that (the vertical bar is read "such that") x has property P.

It may happen that a description fails to describe any number. For example, consider $\{x | x < 2 \text{ and } x > 7\}$. Of course, no number can be simultaneously less than 2 and greater than 7; so this set has no members. We refer to a set with no elements as the **empty set**, or **null set**, and use the special symbol $\varnothing$ to denote it.

Definition of Union and Intersection

The **union** of two sets A and B, denoted $A \cup B$, is the set consisting of all elements that are in A *or* B (or both). See Figure P.6a. The **intersection** of A and B, denoted $A \cap B$, is the set consisting of all elements that are in both A *and* B. In other words, $A \cap B$ consists of the elements common to A and B. See Figure P.6b.

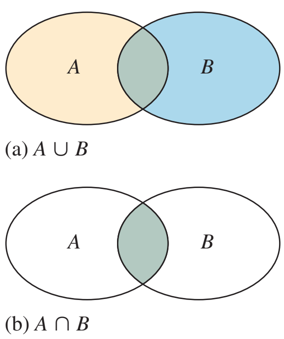


FIGURE P.6 Picturing union and intersection

EXAMPLE 3 Forming Set Unions and Intersections

Find $A \cap B$, $A \cup B$, and $A \cap C$ if $A = \{-2, -1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2, 4\}$, and $C = \{-3, 3\}$

Solution

$A \cap B = \{-2, 0, 2\}$, the set of elements common to both A and B.
$A \cup B = \{-4, -2, -1, 0, 1, 2, 4\}$, the set of elements that are in A or B (or both).
$A \cap C = \varnothing$.

Practice Problem 3 Find $A \cap B$ and $A \cup B$ if $A = \{-3, -1, 0, 1, 3\}$ and $B = \{-4, -2, 0, 2, 4\}$.

4 Use interval notation.

Intervals

We now turn our attention to graphing certain sets of numbers. That is, we graph each number in a given set. We are particularly interested in sets of real numbers, called **intervals**, whose graphs correspond to special sections of the number line.

$(a, b) = \{x | a < x < b\}$

FIGURE P.7 An open interval

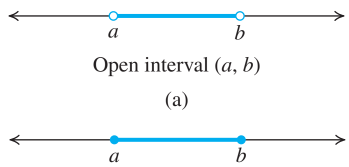
If $a < b$, then the set of real numbers between a and b, but not including either a or b, is called the **open interval** from a to b and is denoted by (a, b). See Figure P.7. Using set-builder notation, we can write

$$(a, b) = \{x | a < x < b\}.$$

We indicate graphically that the endpoints a and b are excluded from the open interval by drawing a left parenthesis at a and a right parenthesis at b. These parentheses enclose the numbers between a and b.

$[a, b] = \{x \mid a \le x \le b\}$

FIGURE P.8 A closed interval



Open interval (a, b)

(a)

Closed interval $[a, b]$

(b)

FIGURE P.9 Endpoint inclusion and exclusion

Interval $(2, \infty)$

FIGURE P.10 An unbounded interval

The **closed interval** from a to b is the set

$$[a, b] = \{x \mid a \le x \le b\}.$$

The closed interval includes both endpoints a and b. We replace the parentheses with square brackets in the interval notation and on the graph. See Figure P.8. Sometimes we want to include only one endpoint of an interval and exclude the other. Table P.1 below shows how this is done.

An alternative notation for indicating whether endpoints are included uses open circles to show exclusion and closed circles to show inclusion. See Figure P.9.

If an interval extends indefinitely in one or both directions, it is called an **unbounded interval**. For example, the set of all numbers to the right of 2,

$$\{x \mid x > 2\},$$

is an unbounded interval denoted by $(2, \infty)$. See Figure P.10.

The symbol ∞ ("infinity") is not a number, but is used to indicate all numbers to the right of 2.

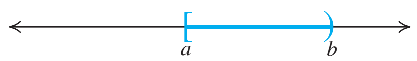
The symbol $-\infty$ is another symbol that does not represent a number. The notation $(-\infty, a)$ is used to indicate the set of all real numbers that are less than a. The notation $(-\infty, \infty)$ represents the set of all real numbers.

Table P.1 lists various types of intervals that we use in this book. In the table, when two points a and b are given, we assume that $a < b$. This is because if $a > b$, then (a, b) is the empty set.

SIDE **NOTE**

The symbols ∞ and $-\infty$ are always used with parentheses, not square brackets. Also note that $<$ and $>$ are used with parentheses and that $\le$ and $\ge$ are used with square brackets.

TABLE P.1

Interval Notation	Set Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\{x \mid x \text{ is a real number}\}$	

EXAMPLE 4 **Union and Intersection of Intervals**

Consider the two intervals $I_1 = (-3, 4)$ and $I_2 = [2, 6]$.

Find: **a.** $I_1 \cup I_2$ **b.** $I_1 \cap I_2$

Solution

a. From Figure P.11, we see that $I_1 \cup I_2 = (-3, 6]$. We note that every number in the interval $(-3, 6]$ is in either I_1 or I_2 or in both I_1 and I_2.

b. We see in Figure P.11 that $I_1 \cap I_2 = [2, 4)$. Every number in the interval $[2, 4)$ is in both I_1 and I_2. Notice that while 4 is in I_2, 4 is not in I_1.

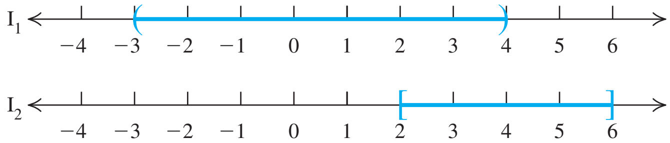


FIGURE P.11

Practice Problem 4 Let $I_1 = (-\infty, 5)$ and $I_2 = [-2, \infty)$. Find the following.

a. $I_1 \cup I_2$ **b.** $I_1 \cap I_2$

5 Relate absolute value and distance on the real number line.

Absolute Value

The ***absolute value*** of a number a, denoted by $|a|$, is the distance between the origin and the point on the number line with coordinate a. The point with coordinate -3 is 3 units from the origin, so we write $|-3| = 3$ and say that the absolute value of -3 is 3. See Figure P.12.

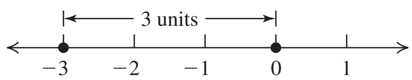


FIGURE P.12 Absolute value

Absolute Value

For any real number a, the **absolute value** of a, denoted $|a|$, is defined by

$$|a| = a \quad \text{if } a \geq 0 \qquad \text{and} \qquad |a| = -a \quad \text{if } a < 0.$$

SIDE **NOTE**

Finding the absolute value requires knowing whether the number or expression inside the absolute value bars is positive, zero, or negative. If it is positive or zero, you can simply remove the absolute value bars. If it is negative, you remove the bars and change the sign of the number or expression inside the absolute value bars.

EXAMPLE 5 Determining Absolute Value

Find the value of each of the following expressions.

a. $|4|$ **b.** $|-4|$ **c.** $|0|$ **d.** $|(-3) + 1|$

Solution

a. $|4| = 4$ Because the number inside the absolute value bars is 4 and $4 \geq 0$, just remove the absolute value bars.

b. $|-4| = -(-4) = 4$ Because the number inside the absolute value bars is -4 and $-4 < 0$, remove the bars and change the sign.

c. $|0| = 0$ Because the number inside the absolute value bars is 0 and $0 \geq 0$, just remove the absolute value bars.

d. $|(-3) + 1| = |-2|$
$= -(-2) = 2$ Because the expression inside the absolute value bars is $(-3) + 1 = -2$ and $-2 < 0$, remove the bars and change the sign.

Practice Problem 5 Find the value of each of the following.

a. $|-10|$ **b.** $|3 - 4|$ **c.** $|2(-3) + 7|$

WARNING

The absolute value of a number represents a distance. Because distance can never be negative, the absolute value of a number is never negative; it is always positive or zero. However, if a is not 0, $-|a|$ is always negative. Thus, $-|5.3| = -5.3$, $-|-4| = -4$, and $-|1.\overline{18}| = -1.\overline{18}$.

Distance Between Two Points on a Real Number Line

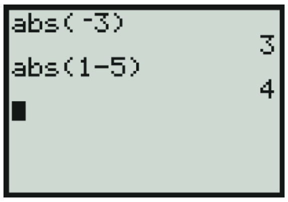
The absolute value is used to define the distance between two points on a number line.

DISTANCE FORMULA ON A NUMBER LINE

If a and b are the coordinates of two points on a number line, then the distance between a and b, denoted by $d(a, b)$, is $|a - b|$. In symbols, $d(a, b) = |a - b|$.

TECHNOLOGY CONNECTION

The absolute value function on your graphing calculator will find the value of the expression entered and then compute its absolute value.



EXAMPLE 6 Finding the Distance Between Two Points

Find the distance between -3 and 4 on the number line.

Solution

Figure P.13 shows that the distance between -3 and 4 is 7 units. The distance formula gives the same answer.

$$d(-3, 4) = |-3 - 4| = |-7| = -(-7) = 7.$$

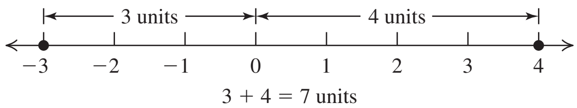


FIGURE P.13 Distance on the number line

Notice that reversing the order of -3 and 4 in this computation gives the same answer. That is, the distance between 4 and -3 is $|4 - (-3)| = |4 + 3| = |7| = 7$. It is always true that $|a - b| = |b - a|$. See Figure P.14.

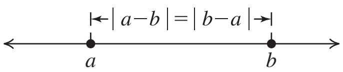


FIGURE P.14

Practice Problem 6 Find the distance between -7 and 2 on the number line.

We summarize the properties of absolute value next.

PROPERTIES OF ABSOLUTE VALUE

If a and b are any real numbers, the following properties apply.

	Property	Example
1.	$\lvert a\rvert \geq 0$	$\lvert -5\rvert = 5$ and $5 \geq 0$
2.	$\lvert a\rvert = \lvert -a\rvert$	$\lvert 3\rvert = \lvert -3\rvert$
3.	$\lvert ab\rvert = \lvert a\rvert\lvert b\rvert$	$\lvert 3(-5)\rvert = \lvert 3\rvert\lvert -5\rvert$
4.	$\left\lvert \frac{a}{b}\right\rvert = \frac{\lvert a\rvert}{\lvert b\rvert}, b \neq 0$	$\left\lvert \frac{-7}{3}\right\rvert = \frac{\lvert -7\rvert}{\lvert 3\rvert}$
5.	$\lvert a - b\rvert = \lvert b - a\rvert$	$\lvert 2 - 7\rvert = \lvert 7 - 2\rvert$
6.	$a \leq \lvert a\rvert$	$-2 \leq \lvert -2\rvert, 3 \leq \lvert 3\rvert$
7.	$\lvert a + b\rvert \leq \lvert a\rvert + \lvert b\rvert$ (the triangle inequality)	$\lvert -2 + 5\rvert \leq \lvert -2\rvert + \lvert 5\rvert$

6 Identify the order of operations in arithmetic expressions.

Arithmetic Expressions

When we write numbers in a meaningful combination of the basic operations of arithmetic, the result is called an **arithmetic expression**. The real number that results from performing all operations in the expression is called the **value** of the expression. In arithmetic and algebra, parentheses ( ) are **grouping symbols** used to indicate which operations are to be performed first. Other common grouping symbols are square brackets [ ], braces { }, fraction bars — or /, and absolute value bars | |.